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Reference points under the hypothesis of a sex-specific life-history [1]

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Summary

The detailed description of several reference points under a sex-specific model was proposed and were applied to North Pacific albacore based on the stock assessment in 2014. Commonly confused settings of reference points which come from sex-specific properties were pointed out. providing a material for further discussion towards the next stock assessment in 2017. R source code that was used for this document was attached.

1 Introduction

In the current stock assessment of North Pacific albacore (*Thunnus alalunga*) (NPALB). a two-sex growth model was used in order to better fit the size data showing a different size distribution between sexes (ISC 2014). Following a conventional assessment, a series of reference points (RPs) that indicates the state of biomass and fishery intensity was provided. but the definitions of RPs under a two-sex growth model has not been discussed yet. Although the software Stock Synthesis 3 (SS3) (Methot and Wetzel 2013), that is used for the stock assessment, can handle these RPs, the detailed process is not clear for the great majority of users.

Recently, introduction of management strategy evaluation (MSE) framework into the assessment of NPALB has been well discussed for several reasons (NC11 2015), resulting in that RPs, as well as operational objectives and decision rules, would play a central role than before. Thus, among scientists responsible for the assessment, understanding as to how sex-specific assumptions affect the RPs becomes important.

In this document, we firstly provide a definition of the RPs under a sex-specific model. Here, we adopted a more simple approach to calculate RPs for keeping the tractability. Then, we apply them to NPALB based on the stock assessment in 2014. Finally, we discuss the interpretation of RPs and provide some caveats related to sex-specific assumptions.

2 Method

Table 1 summarizes all symbols used in this document.

2.1 Overview

Under an age-structured model, the definition of RPs based on fishing mortality (i.e., F-based RPs) is a bit complex and may not be shared among fishery scientists. Thus, first we provide a way of calculating them as a generic form. Then, sex-specific issues involved to RPs are descrived.

Fundamental assumption of F-based RPs is that selectivity of fishing gear is identical to that of target year(s) which are selected arbitrarily (usually current year(s) and/or reference year(s) are used). The value of a ratio of $F_{\text{target},a}$ to F_a that satisfies desired property, such as MSY, is usually referred to as RPs. It should be noted that, depending on regional fisheries management organizations, the ratio is inversely defined. For explanation, suppose that $F_{\text{curent}}/F_{\text{MSY}} = 0.5$. This states: "as long as the current selectivity holds, doubling the intensity of fishery leads to F_{MSY}." Here, we call the multiplied value (i.e., "two" in this example) F-multiplier, that is denoted by a scalar f. In practice, given target year(s), the ratio can be numerically obtained by changing f. This way of calculation allows a variable intensity of fishing but requires a fixed-selectivity, leading to some problems in a sex-specific model, as explained later. Figure 1 demonstrates these points by taking NPALB as a example. Detailed description of RPs under standard models is well documented in Caddy and Mahon (1995) and Quinn and Deriso (1999), but their relevance to a sex-specific model is little documented.

2.1.1 Fishing mortality at age, F_a

Instantaneous fishing mortality rate at age (denoted by $F_{a,year}$) is not supported by the current version of SS3, so we obtained $F_{a,year}$ from catch at age ($C_{a,year}$), number at age ($N_{a,year}$) and natural mortality at age (M_a) by numerically solving the Baranov catch equation:

$$C_{a,year} = \frac{F_{a,year}}{F_{a,year} + M_a} \exp[-F_{a,year} - M_a] N_{a,year}. \tag{1}$$

For the purpose of simplicity, quarter-based catch are aggregated into year-based, such as $C_{a,year} = C_{a,year,q=1} + C_{a,year,q=2} + C_{a,year,q=3} + C_{a,year,q=4}$ in the case that the spawning season is assumed in beginning of quarter 1.

If the reference term is multi-year, $F_{\text{target},a}$ was obtained by averaging $F_{a,year}$ among years, such as $F_{2010-2012}$. Here, arithmetical average is used.

$$F_{\text{target},a} = \frac{1}{\text{Number of target years}} \sum_{year=\text{StartYear}}^{\text{EndYear}} F_{a,year}. \tag{2}$$

2.1.2 Weight at age, W_a

For simplicity, two assumptions are made about the weight at age (denoted by W_a). First, we consider the quarter when the spawning occurs as a representative term for calculating RPs, although the weigh of fish must vary by quarters and fisheries are operated primarily during several quarters. Second, increasing of weight as age is assumed to be stopped at a max age and the weight is considered as that of a plus-group. The latter may be justified that there are little fishes with extreme old age.

2.1.3 Sex-specific model

So far, we ignore sex-specific issues, although SS3 can handle sex-specific properties including sex ratio (ratio of $N_{♂ a=0}$ to $N_{♀♂ a=0}$) and produce separate estimations by sex (e.g., catch, number, weight). This brings distinctive fishing mortality by sex, as shown in Fig. 1. In this case, definition of several variables may be changed, thus we carefully check the treatment of sex-specific properties. For example, in NPALB, spawning-stock-biomass (SSB) is composed by *female* individuals, therefore the stock-recruitment relationship ignores male biomass. This setting would affect the definition of RPs. We should recognize these treatments especially in applying to MSE (or harvest control rule) processes.

2.2 Definition of reference points

Table 2 summarizes all RPs noted in this document. R-code that calculates RPs is in Appendix.

2.2.1 YPR (Yield-Per-Recruit) criteria, $F_{0.1}$

YPR analysis has been used for maximizing the yield from recruits by avoiding growth overfishing, but does not take into account the effect of fishing mortality on the reproductive potential due to lack of information of maturity and thus SSB. This keeps the relatively small modification of the definition of YPR under a sex-specific model; YPR contains the information of both female and male weighted by sex ratio (recruitment does not distinguish sexes), defined by

$$YPR_{\text{target}}(f) = (1-r)\sum_{a=0}^{a_{\max}}\left[\mu_{♀,a}(f)\times\lambda_{♀,a}(f)\times W_{♀,a}\right] + r\sum_{a=0}^{a_{\max}}\left[\mu_{♂,a}(f)\times\lambda_{♂,a}(f)\times W_{♂,a}\right]. \quad (3)$$

where

$$\mu_{♀,a}(f) = \frac{fF_{\text{target},♀,a}}{fF_{\text{target},♀,a}+M_{♀,a}}(1-\exp[-fF_{\text{target},♀,a}-M_{♀,a}]), \quad (4)$$

$$\lambda_{♀,a}(f) = \prod_{a'=0}^{a-1}\exp[-fF_{\text{target},♀,a'}-M_{♀,a'}], \quad (5)$$

$$\mu_{♂,a}(f) = \frac{fF_{\text{target},♂,a}}{fF_{\text{target},♂,a}+M_{♂,a}}(1-\exp[-fF_{\text{target},♂,a}-M_{♂,a}]), \quad (6)$$

$$\lambda_{♂,a}(f) = \prod_{a'=0}^{a-1}\exp[-fF_{\text{target},♂,a'}-M_{♂,a'}]. \quad (7)$$

μ and λ means exploitation fraction and cumulative survival, respectively. While μ_a shows dome-shaped curve with f, λ_a exponentially decreases, thus it is expected that there exists f such that YPR has maximum.

From Eq. 3, we can obtain $f_{0.1}$ that satisfies

$$\left.\frac{YPR_{\text{target}}(f)}{df}\right|_{f=f_{0.1}} = 0.1\left.\frac{YPR_{\text{target}}(f)}{df}\right|_{f=0}. \tag{8}$$

$f_{0.1}$ is the point on the YPR curve where the slope of the curve is 10% of the one at the origin, as illustrated in Fig. 2. Following our notation, $1/f_{0.1}$(corresponding to $F_{\text{target}}/F_{0.1}$) is RP ratio of $F_{0.1}$.

2.2.2 SPR (Spawning-Per-Recruit) criteria, $F_{\%SPR}$

In contrast to $F_{0.1}$ based on YPR, there are several RPs that consider reproductive potential, such as $F_{\%SPR}$, F_{MSY} and F_{MED}. SPR that is an extension of YPR can incorporate maturity, defined by

$$SPR_{\text{target}}(f) = (1-r)\sum_{a=0}^{a_{\max}}\left[Q_{♀,a}\times\lambda_{♀,a}(f)\times W_{♀,a}\right]. \tag{9}$$

SPR exponentially decreases as f increases, as shown in Fig. 3. In a sex-specific model, there might be some confusing points. Since SSB does not contain male individuals, SPR is determined only from the female information. By definition, $SSB_{♀} = SPR_{target}R$ must be satisfied, so the term $(1-r)$, female ratio, is required (recruitment contains both sexes). It should be noted that SPR is a mapping from R to $SSB_{♀}$ (see also Fig. 4A).

From Eq. 9, we can obtain $f = f_{\alpha\%}$ that satisfies

$$\frac{SPR_{\text{target}}\big|_{f=f_{\alpha\%}}}{SPR_{\text{target}}\big|_{f=0}} = \frac{\alpha}{100}; \tag{10}$$

and RP ratio of $F_{\alpha\%}$ is $1/f_{\alpha\%}$(corresponding to $F_{\text{target}}/F_{\alpha\%}$). Left-hand side of Eq. 10 is scaled SPR by that with no fishing; when $f = 0$, the left-hand side equals to one.

$F_{\%SPR}$ is a reference fishing mortality that results in a *female* SSB (or egg production) per recruitment that is α% of that with no fishing.

2.2.3 MSY (Maximum-Sustainable-Yield) criteria, F_{MSY}

MSY is a concept that aims to maintain the population size at the point of maximum growth rate and thus maximum surplus yield. MSY is calculated by assuming equilibrium, so it must require a careful interpretation (summarized in Caddy and Mahon 1995)! At equilibrium, *female* sustainable yield in a sex-specified model, $SY_{♀}$, can be defined by,

$$SY_{♀}(f) = R_{\text{eq}}(f)(1-r)\sum_{a=0}^{a_{\max}}\left[\mu_{♀,a}(f)\times\lambda_{♀,a}(f)\times W_{♀,a}\right]. \tag{11}$$

Subscript $_{eq}$ indicates the state variable at equilibrium. Given f, R_{eq} satisfies

$$R = g(SSB_♀)SSB_♀. \quad (12)$$

$$SSB_♀ = SPR_{target}R. \quad (13)$$

where the right-hand side of Eq. 12 indicates a relationship between *female* SSB and recruitment. Figure 4A illustrates $(SSB_{♀,eq}, R_{eq})$ for various f under the B-H relationship.

From Eq. 11, we can obtain $f = f_{MSY}$ that maximizes $SY_♀$ (shown in Fig. 4B), and RP ratio of F_{MSY} is $1/f_{MSY}$(corresponding to F_{target}/F_{MSY}).

Alternatively, sustainable yield of *both sexes* can be defined by,

$$SY_{♀♂}(f) = R_{eq}(f)YPR_{target}(f). \quad (14)$$

Despite of this simple expression, the interpretation of $SY_{♀♂}$ is a bit tricky: the equilibrium point is independent of males, but YPR of males affects the sustainable yield, possibly leading to different f_{MSY} defined in Eq. 11. This complex definition (Eq. 14) may be confusing, so we adopted the former one in this document.

2.2.4 RPS (Yield-Per-Recruit) criteria, F_{MED}

Calculation of MSY requires the S-R relationship (function $g()$ in Eq. 12), suggesting that uncertainty of the estimation would be crucial. Without applying S-R model, RPS analysis assumes a linear-mapping from $SSB_♀$ to R through the origin. In the case that, based on a S-R data set, the RPS-line has 50% of the recruitment above and 50% below, the slope of the line can be defined as median RPS (denoted by RPS_{MED}) in the history, as shown in Fig. 5A. For a population at equilibrium, fishing mortality that can achieve RPS_{MED} would keep the population stable at the median level (Quinn and Deriso 1999).

Following relationships are satisfied at equilibrium,

$$R(f) = RPS_{MED}SSB_♀(f). \quad (15)$$

$$SSB_♀(f) = SPR_{target}(f)R(f). \quad (16)$$

By removing $SSB_♀/R$, we obtain

$$SPR_{target}(f) = \frac{1}{RPS_{MED}} \quad (17)$$

and we can obtain $f = f_{MED}$ that satisfies Eq. 17. Figure 5B illustrates this relationship. RP ratio of F_{MED} is $1/f_{MED}$(corresponding to F_{target}/F_{MED}).

2.3 Application to North Pacific Albacore

The setting of parameters and assumptions are same as the stock assessment of NPALB in 2014 (ISC 2014). Relevant items of this document are as follows:

- sex ratio is 1:1 (i.e., $r = 0.5$).
- sex-specific growth model.
- SSB is defined by a female spawning-stock-biomass.
- recruitment is defined by female and male number of fish at age zero.
- spawning season is a second quarter.
- catch is summed from the second quarter to the first quarter in next year.
- 2002-2004 and 2010-2012 are selected as target years for calculating RPs.
- $C_{a,2012} = C_{a,2012,q=2} + C_{a,2012,q=3} + C_{a,2012,q=4}$ because $C_{a,2013,q=1}$ is not available.
- S-R relationship was B-H type with $h = 0.9$, as

$$R_{year+1} = \frac{4hR_0 SSB_{♀,year}}{SSB_{♀,0}(1-h) + SSB_{♀,year}(5h-1)}. \tag{18}$$

3 Results and Discussion

3.1 RPs of NPALB

Figure 1 shows $F_{\text{target},a}$ of female and male distinctively. Between 2010-2012, the peak of fishing mortality is shifted toward older fishes, especially in males, than between 2002-2004. Moreover, between 2010-2012, the remarkable difference between sexes was found in older ages.

Table 3 shows RP ratios for various RPs which are noted in this document. The value of RP ratio is an inverse of F-multiplier, f, that satisfies the corresponding property as noted in "Method" section. When the ratio is less than one, the pattern of fishing (i.e., selectivety) in target years can theoretically achieve the corresponding criteria. Due to the realization of the shift towards older fishes, $F_{2010-2012}$ leads to better situations than $F_{2002-2004}$.

3.2 Caveats

Here, we provide several caveats when using the RPs under a sex-specific model. Male individuals are almost ignored in determination of RPs except YPR criteria because SSB is usually defined by females, suggesting that these RPs do not provide the information on fishing of males if both fishing patterns are independently determined. For example, it is not necessarily that F_{MSY} (derived from Eq. 11) leads to MSY that includes both sexes.

3.3 Future works

Followings are needed to be checked before applying proposed definitions of RPs to the next stock assessment:

- consistency with SS outputs,
- accuracy of approximation we used in practical situations, such as the case of NPALB.

Reference

Caddy, J. F. and R. Mahon, 1995. Reference points for fisheries management, volume 374. Food and Agriculture Organization of the United Nations Rome.

ISC, 2014. Stock assessment of albacore tuna in the north pacific ocean in 2014. In *Annex 11 of the International Scientific Committee for Tuna and Tuna-like Species in the North Pacific Ocean Plenary Report.*

Methot, R. D. and C. R. Wetzel, 2013. Stock synthesis: a biological and statistical framework for fish stock assessment and fishery management. *Fisheries Research* **142**:86–99.

NC11, 2015. Proposed framework for management strategy evaluation for north pacific albacore tuna. In *11th Regular Session of the Northern Committee.*

Quinn, T. J. and R. B. Deriso, 1999. Quantitative fish dynamics. Oxford University Press.

Table 1: List of mathematical symbols

Age-structured variables	
$M_{♀,a}, M_{♂,a}$	Natural mortality at age
$F_{♀,a,year}, F_{♂,a}$	Fishing mortality at age
$F_{target,♀,a}, F_{target,♂,a}$	Arithmetic mean of fishing mortality among target years
$W_{♀,a}, W_{♂,a}$	Weight at age
$\lambda_{♀,a}, \lambda_{♂,a}$	Cumulative survival
$\mu_{♀,a}, \mu_{♂,a}$	Exploitation fraction
$C_{♀,a}, C_{♂,a}$	Catch at age
$N_{♀,a}, N_{♂,a}$	Number at age
$Q_{♀,a}$	Maturity at age
State variables	
$R(= N_{a=0})$	Number of recruitments (including both sexes)
$SY_{♀}$	*female* sustainable yield
$SSB_{♀}$	*Female* spawning-stock-biomass
Parameters	
r	sex ratio at birth (i.e., $N_{♂,a=0} : N_{♀,a=0} = r : (1-r)$)
f	F-multiplier searching for RPs
R_0	Unfished equilibrium of recruitment (including both sexes)
$SSB_{♀,0}$	Unfished equilibrium of *female* spawning-stock-biomass
h	Steepness

Table 2: Summary of reference points

	$F_{0.1}$	$F_{\%SPR}$	F_{MSY} (Eq. 11)	F_{MED}
Involved sex of adults for derivation	♀♂	♀	♀	♀
Assuming equilibrium	No	No	Yes	Yes
Required information	M_a, W_a	M_a, W_a, Q_a	M_a, W_a, Q_a, S-R relationship	M_a, W_a, Q_a, S-R data

Table 3: Reference point ratio

	$F_{2010-2012}$	$F_{2002-2004}$
$F_{0.1}$	0.53	0.63
$F_{10\%}$	0.32	0.45
$F_{20\%}$	0.48	0.66
$F_{30\%}$	0.66	0.90
$F_{40\%}$	0.89	1.20
$F_{50\%}$	1.21	1.60
F_{MSY}	0.46	0.63
F_{MED}	1.06	1.41

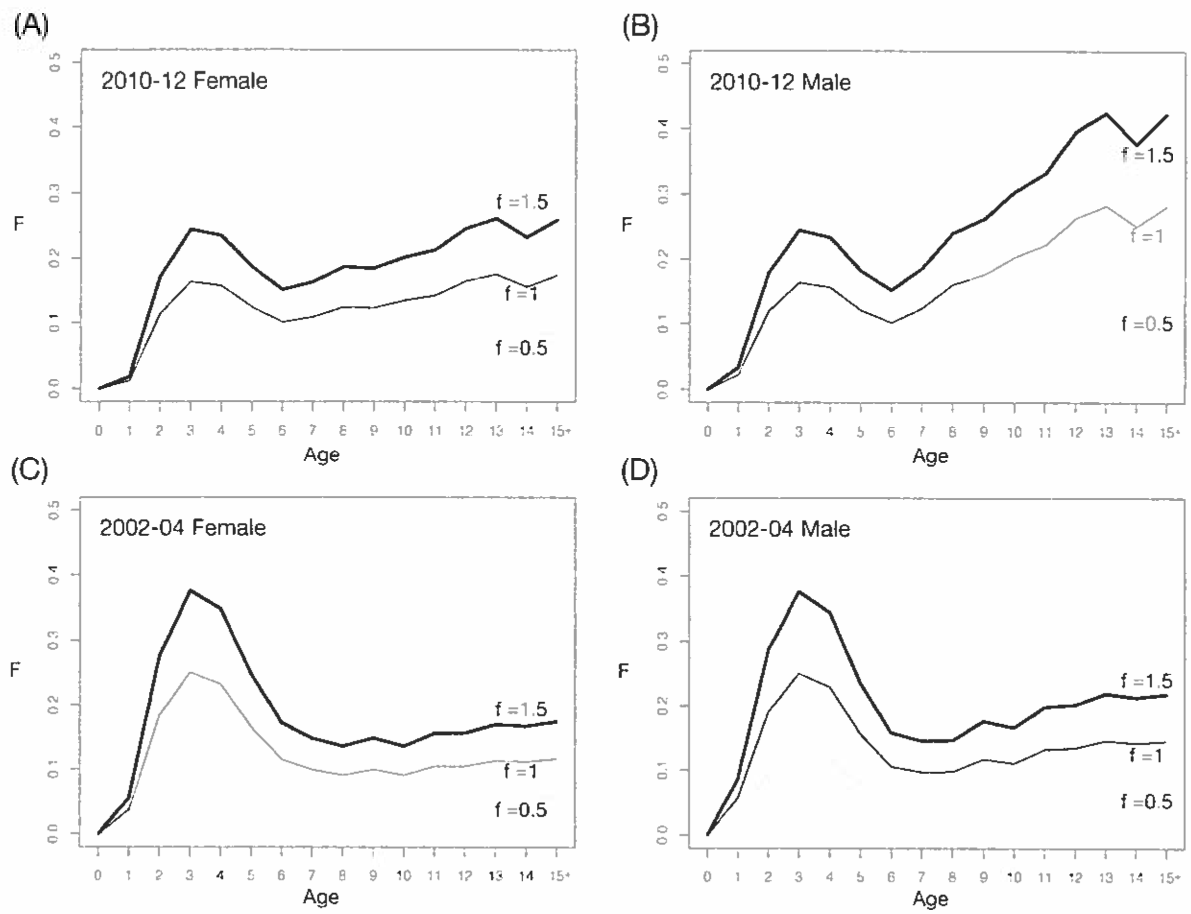

Figure 1: Female and male fishing mortality of NPALB between 2010 and 2012 (A-B), between 2002 and 2004 (C-D). RPs are obtained by changing F-multiplier (f), suggesting the selectivity is fixed in the target years. $f = 1.5$ and $f = 0.5$ are shown for the illustrative purpose.

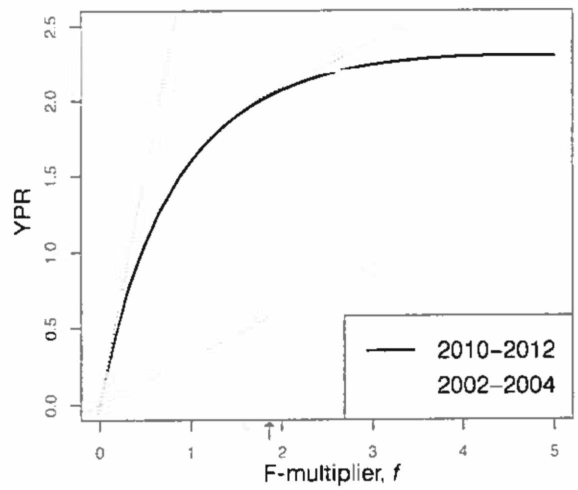

Figure 2: YPR curves of NPALB. Arrows indicate $f_{0.1}$. Auxiliary dotted lines for getting $f_{0.1}$ are corresponding to the target years 2010-2012. Gray curve is plotted in the range of (0,3.6) because a larger value of f leads to an extinction of $SSB_{♀}$ at equilibrium.

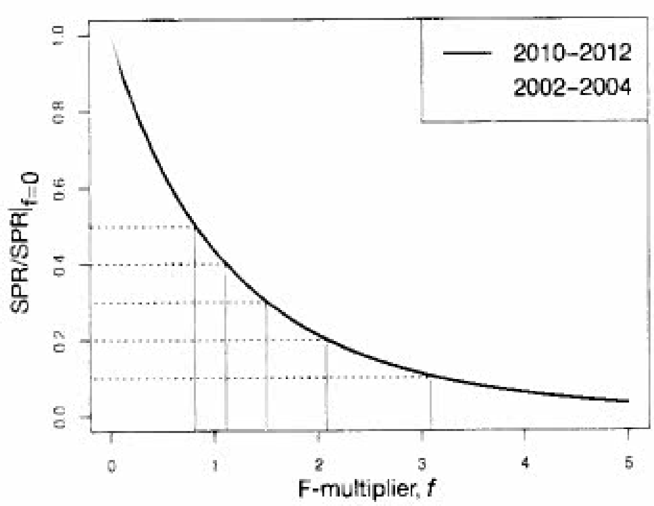

Figure 3: SPR curves of NPALB (scaled by $\text{SPR}|_{f=0}$). The position of the intersection between horizontal axis and lines corresponds to $f_{\%SPR}$ ($f_{50\%}$, $f_{40\%}$, $f_{30\%}$, $f_{20\%}$ and $f_{10\%}$).

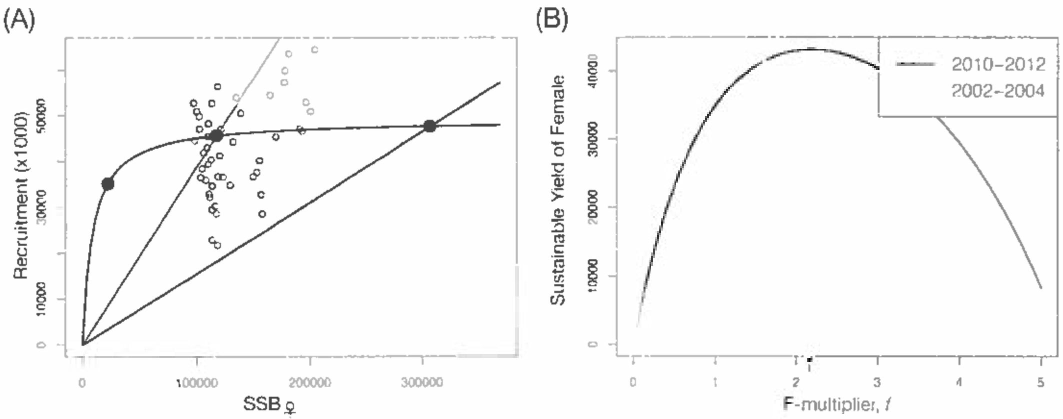

Figure 4: (A) S-R relationship and SRP lines (2010-2012) of NPALB. Filled circles indicate equilibrium points ($SSB_{♀,eq}, R_{eq}$). Right equilibrium point that is an intersection to $SPR_{|f=0}$ line shows ($SSB_{♀,0}, R_0$). Middle and left point is corresponding to $f_{40\%}$ and $f_{10\%}$, respectively. (B) Female sustainable yield defined at equilibrium. Arrows indicate f_{MSY}.

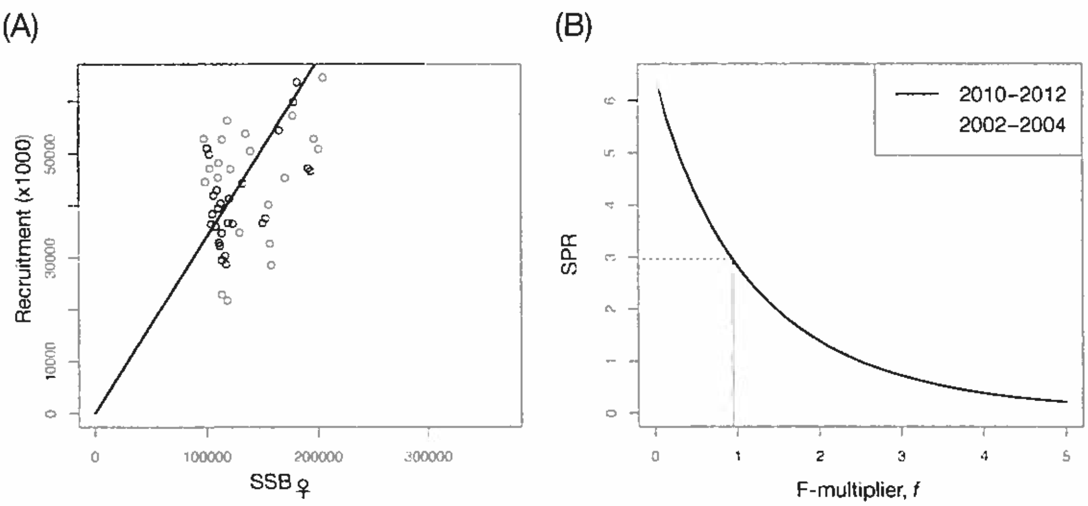

Figure 5: (A) RPS_{MED} line of NPALB. The slope (corresponding to RPS_{MED}) is 0.34 and the inverse is 2.94. (B) SPR curve of NPALB. The position of the intersection between horizontal axis and lines corresponds to f_{MED}

R-code 1: Personally used source code in this document

```
# Get Reference Points, developed by Tetsuya Akita

# INPUT PARAMETERS
mydir <- "~/Seek/Simplicity/But/Distrust/It" # include related files in this folder
tgtyr_start <- 2010 # target year for refferenced F. (e.g., F0204)
tgtyr_end <- 2012
nbin_F <- 10000
MaxFmult <- 5

# Read SS outputs
setwd(mydir)
library(r4ss)
res <- SS_output(dir = mydir, model = "ss3", repfile = "Report.sso",compfile = "CompReport.sso",
    covarfile = "covar.sso", ncols = 330, forecast = TRUE, warn = TRUE, covar = F, checkcor = TRUE,
    cormax = 0.95,cormin = 0.01,printhighcor = 10,printlowcor = 10, verbose = TRUE, printstats = TRUE
    , hidewarn = FALSE,NoCompOK = FALSE,aalmaxbinrange = 4)

MaxAge <- res$accuage
StartYear <- res$startyr
EndYear <- res$endyr

# Check the number of gender
ngender <- max(res$morph_indexing$Gender)
cat("ngender=",ngender,"\n")

# Get biological parameters, M@A, W@A, Q@A
bio <- {}
tmp = res$endgrowth #Wt_Beg=waa in season 2
bio[[1]] = tmp[which((tmp$Seas==2)&(tmp$Gender==1)),names(tmp)%in%c("M","Wt_Beg","Age_
    Mat")]
bio[[2]] = tmp[which((tmp$Seas==2)&(tmp$Gender==2)),names(tmp)%in%c("M","Wt_Beg")]
rownames(bio[[1]]) <- rownames(bio[[2]]) <- tmp$Age_Beg[which((tmp$Seas==2)&(tmp$Gender
    ==1))]

M1 <- bio[[1]]$M
M2 <- bio[[2]]$M
W1 <- bio[[1]]$Wt_Beg
W2 <- bio[[2]]$Wt_Beg
Q <- bio[[1]]$Age_Mat

rm(tmp)

# Get S-R relationship
SSB <- res$recruit$spawn_bio[res$recruit$year>=StartYear]
REC <- res$recruit$pred_recr[res$recruit$year>=StartYear]
R0 <- sum(res$timeseries$Recruit_0[res$timeseries$Era == "VIRG"], na.rm = TRUE)
```

```r
B0 <- sum(res$timeseries$SpawnBio[res$timeseries$Era == "VIRG"], na.rm = TRUE)
h <- res$parameters$Value[res$parameters$Label%in%c("SR_BH_steep","SR_BH_flat_steep")]
cat("R0 =",R0,", B0_female =",B0,", h =",h,"\n")

# Get C@A
tmp <- res$catage
cat("Number of fleet =", max(tmp$Fleet),"\n" )
qY <- (max(tmp$Yr) - min(tmp$Yr) +1 ) * 4 # = (2012 - 1965 + 1)*4 = 48*4
nyear <- qY/4 # = 48, 1965 to 2012, first qs and last three qs are removed
cat("Start term of CAA is year:",min(tmp$Yr)+1,", season:", 2,"\n")
cat("End term of CAA is year:",max(tmp$Yr),", season:", 1,"\n")
cat("catch in",max(tmp$Yr),"is included into year",max(tmp$Yr)-1,"\n")
cat("CAA has nrow =",nyear,"\n")

CAA1 <- CAA2 <- matrix(0, nyear, MaxAge+1)

# NOTE: last year (2012) is sumed only by three seasons
for (i in 1:nyear){
  # CAA female
  CAA1[i,] <- apply(tmp[(tmp$Yr==(min(tmp$Yr)+i-1)) & (tmp$Seas%in%c(2,3,4)) &(tmp$Gender==1),][names(tmp)%in%seq(0,MaxAge)], 2, sum)
  if(i!=nyear) CAA1[i,] <- CAA1[i,] + apply(tmp[(tmp$Yr==(min(tmp$Yr)+i)) & (tmp$Seas==1) &(tmp$Gender==1),][names(tmp)%in%seq(0,MaxAge)], 2, sum)
  # CAA male
  CAA2[i,] <- apply(tmp[(tmp$Yr==(min(tmp$Yr)+i-1)) & (tmp$Seas%in%c(2,3,4)) &(tmp$Gender==2),][names(tmp)%in%seq(0,MaxAge)], 2, sum)
  if(i!=nyear) CAA2[i,] <- CAA2[i,] + apply(tmp[(tmp$Yr==(min(tmp$Yr)+i)) & (tmp$Seas==1) &(tmp$Gender==2),][names(tmp)%in%seq(0,MaxAge)], 2, sum)
}
rownames(CAA1) <- rownames(CAA2) <- seq(min(tmp$Yr),max(tmp$Yr))
colnames(CAA1) <- colnames(CAA2) <- seq(0,MaxAge)
rm(tmp)

# GetN@A
tmp <- res$natage
tmp1 <- tmp[which( (tmp$Seas==2) & (tmp$`Beg/Mid`=="B") & (tmp$Yr >= (StartYear-1)) & (tmp$Yr <= EndYear) & (tmp$Gender==1) ) ,]
tmp2 <- tmp[which( (tmp$Seas==2) & (tmp$`Beg/Mid`=="B") & (tmp$Yr >= (StartYear-1)) & (tmp$Yr <= EndYear) & (tmp$Gender==2) ) ,]
NAA1 <- as.matrix(tmp1[names(tmp1)%in%seq(0,MaxAge)])
NAA2 <- as.matrix(tmp2[names(tmp2)%in%seq(0,MaxAge)])
rownames(NAA1) <- rownames(NAA2) <-dimnames(CAA1)[[1]]
rm(tmp)

# GetF@A
nage <- MaxAge+1
FAA1 <- FAA2 <- matrix(0, nyear, nage)
```

```
for (i in 1:nyear){
  for (j in 1:nage){
    if (NAA1[i,j]>0.0000001) {
      F0 <- CAA1[i,j] / NAA1[i,j]
      F1 <- CAA1[i,j] * (F0 + M1[j]) / NAA1[i,j] / (1 - exp(-F0-M1[j]))
      while(abs(F0-F1)>0.0001){
        F0 <- F1
        F1 <- CAA1[i,j] * (F0 + M1[j]) / NAA1[i,j] / (1 - exp(-F0-M1[j]))
      }
      FAA1[i,j] <- F1
    }
  }
}
for (i in 1:nyear){
  for (j in 1:nage){
    if (NAA2[i,j]>0.0000001) {
      F0 <- CAA2[i,j] / NAA2[i,j]
      F1 <- CAA2[i,j] * (F0 + M2[j]) / NAA2[i,j] / (1 - exp(-F0-M2[j]))
      while(abs(F0-F1)>0.0001){
        F0 <- F1
        F1 <- CAA2[i,j] * (F0 + M2[j]) / NAA2[i,j] / (1 - exp(-F0-M2[j]))
      }
      FAA2[i,j] <- F1
    }
  }
}

rownames(FAA1) <- rownames(FAA2) <- dimnames(CAA1)[[1]]
colnames(FAA1) <- colnames(FAA2) <- dimnames(CAA1)[[2]]

# Get target F@A as a arithmetical-mean vector
tmp <- FAA1[rownames(FAA1)%in%seq(tgtyr_start,tgtyr_end),]
TargetFAA1 <- apply(tmp, 2, function(x) {
  GeoMean <- mean(x)#exp(mean(log(x)))
})
tmp <- FAA2[rownames(FAA2)%in%seq(tgtyr_start,tgtyr_end),]
TargetFAA2 <- apply(tmp, 2, function(x) {
  GeoMean <- mean(x)#exp(mean(log(x)))
})
plot(NULL,pch=NA,xlim=c(0,MaxAge),ylim=c(0,0.3),xlab="",ylab="",cex.axis=1.2,cex.main=1.0,
     main = "F1012,-N:Q2,-Catch:Q2-3-4-1",xaxp=c(0, 15, 15) )
grid(NULL)
points(seq(0,MaxAge),TargetFAA1,type = "b",col="red")
points(seq(0,MaxAge),TargetFAA2,type = "b",col="blue")

# Get RPs

# NOTE: if SSB goes to zero, calculation of RPs is stopped!
```

```
N1 <- N2 <- rep(1,MaxAge+1)
Fmult <- seq(0,MaxFmult,length=nbin_F)
SPR <- YPR <- SSBmsy <- Rmsy <- MSY <- rep(0,length=nbin_F)

for(i in 1:length(Fmult)) {
  FAAmulti1 <- TargetFAA1*Fmult[i]
  FAAmulti2 <- TargetFAA2*Fmult[i]
  Z1 <- FAAmulti1 + M1
  Z2 <- FAAmulti2 + M2
  for(j in 1:MaxAge){
    N1[j+1] <- N1[j]*exp(-Z1[j])
    N2[j+1] <- N2[j]*exp(-Z2[j])
  }
  N1[MaxAge+1] <- N1[MaxAge]*exp(-Z1[MaxAge])/(1-exp(-Z1[MaxAge+1]))
  N2[MaxAge+1] <- N2[MaxAge]*exp(-Z2[MaxAge])/(1-exp(-Z2[MaxAge+1]))

  Catch_ratio1 <- FAAmulti1/Z1*(1-exp(-Z1))
  Catch_ratio2 <- FAAmulti2/Z2*(1-exp(-Z2))
  YPR[i] <- 0.5*sum(N1*W1*Catch_ratio1) + 0.5*sum(N2*W2*Catch_ratio2)
  SPR[i] <- 0.5*sum(N1*W1*Q)

  if(i==1) {
    spr0 <- SPR[i]
  }else{ # NOTE: SPR under F=0 is ignored when getting equilibliun_S&R
    spr <- SPR[i]
    f <- function (x) 4*h*R0*x / ( spr0*R0*(1-h) + x*(5*h-1) ) - x/spr
    # obtein intersection between SR-curve & RPS-line
    sol <- uniroot(f, c(0.1, 100*B0))
    SSBmsy[i] <- sol$root
    Rmsy[i] <-SSBmsy[i]/spr
  }
  MSY[i] <- 0.5*sum(N1*W1*Catch_ratio1)*Rmsy[i] # Equation in 11
  # MSY[i] <- YPR[i]*Rmsy[i] # Equation in 14
}

# YPR series
plot(Fmult,YPR,type = "l")
grad_YPR <- (YPR[2:nbin_F]-YPR[1:(nbin_F-1)])/(MaxFmult/nbin_F)
Fmax <- 1/(Fmult[grad_YPR<0])[1]
grad_YPR0 <- 0.1*grad_YPR[1]
F0.1 <- 1/(Fmult[grad_YPR<grad_YPR0])[1]
cat("Fmax=",Fmax,",F0.1=",F0.1,"\n")

# SPR series
SPR0 <- SPR/SPR[1] # scaling so that SPR with F=0 equals one
plot(Fmult,SPR0,type = "l")
F50 <- 1/(Fmult[SPR0<0.5])[1]
```

```
F40 <- 1/(Fmult[SPR0<0.4])[1]
F30 <- 1/(Fmult[SPR0<0.3])[1]
F20 <- 1/(Fmult[SPR0<0.2])[1]
F10 <- 1/(Fmult[SPR0<0.1])[1]
cat("F50%=",F50,"F40%=",F40,",F30%=",F30,",F20%=",F20,",F10%=",F10,"\n")


# SPR and S-R plot
options(scipen=1)
plot(NULL, xlim = c(0,1.2*B0),ylim = c(0,max(REC)), xlab = "", ylab = "",cex.axis=0.7 )
points(SSB,REC)
curve(1/SPR[1]*x,add = T,lwd=2)
curve(1/SPR[SPR0<0.4][1]*x ,add = T)
curve(1/SPR[SPR0<0.1][1]*x ,add = T,col="grey")
curve(4*h*R0*x / ( spr0*R0*(1-h) + x*(5*h-1) ),add = T,lwd=2)

# MSY
plot(Fmult,MSY,type = "l",ylab = "Yield")
grad_MSY <- (MSY[2:nbin_F]-MSY[1:(nbin_F-1)])/(MaxFmult/nbin_F)
Fmsy <- 1/(Fmult[grad_MSY<0])[1]
Bmsy <- (SSBmsy[grad_MSY<0])[1]
msy <- (MSY[grad_MSY<0])[1]
cat("Fmsy=",Fmsy,",SSBmsy=",floor(Bmsy),",MSY=",floor(msy),"\n")

# RPS series
plot(NULL, xlim = c(0,B0),ylim = c(0,max(REC)), xlab = "SSB", ylab = "REC" )
points(SSB,REC)
curve(4*h*R0*x / ( spr0*R0*(1-h) + x*(5*h-1) ),add = T,col="red")

num <- EndYear-StartYear+1
num10 <- floor(0.1*num)
num50 <- floor(0.5*num)
num90 <- floor(0.9*num)
g <- function(x) {
  RPSS <- SSB*x
  sum((REC-RPSS)<0)
}
RPS <- seq(from=0.1,to = 1,by = 0.01)
tmp <- sapply(RPS,g)

RPS_l <- (RPS[tmp>num10])[1]
RPS_m <- (RPS[tmp>num50])[1]
RPS_h <- (RPS[tmp>num90])[1]

curve(RPS_l*x,add = T,lty=2)
curve(RPS_m*x,add = T,lty=2)
curve(RPS_h*x,add = T,lty=2)
```

```
plot(Fmult,SPR,type = "l")
F_l <- 1/(Fmult[SPR<1/RPS_l])[1]
F_m <- 1/(Fmult[SPR<1/RPS_m])[1]
F_h <- 1/(Fmult[SPR<1/RPS_h])[1]
cat("F_low = ",F_l,", F_mid = ",F_m,", F_high = ",F_h,"\n")
```