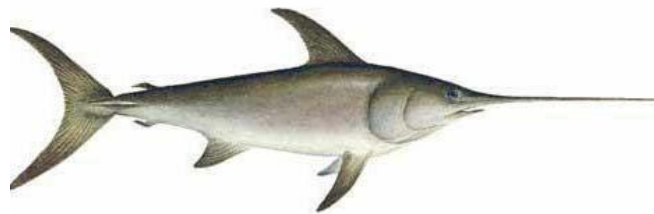
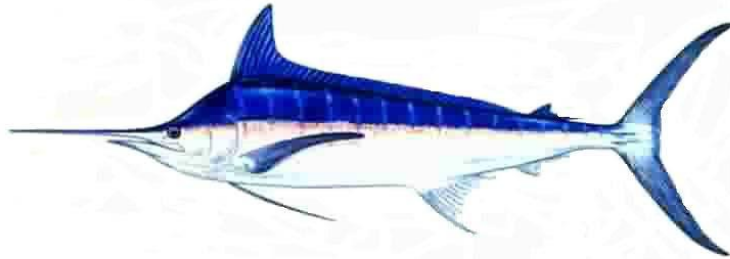




## Combining Information on Length-Weight Relationships for Pacific Blue Marlin<sup>1</sup>

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### Abstract

In this working paper, I provide a meta-analysis of the available studies that provide standard information on the allometric model of weight as a function of length for female, male, and combined-sex Pacific blue marlin (Wang et al. 2006, Su et al. 2012, Sun et al. 2012). The meta-analysis treated the parameters of the allometric model  $BW = A \cdot EFL^B$  relating body weight ( $BW$ , kg) to eye-fork length ( $EFL$ , cm) as effect sizes and combined this information to estimate the mean effect size using the random-effects model. Sample sizes of the available studies were used to weight the effect sizes under a simplifying assumption of homogeneous within-study variances and a rough approximation of the order of magnitude of the between-studies variance. Random effects meta-analysis results indicated that the mean effects for females were  $A = 1.844 \cdot 10^{-5}$  and  $B = 2.956$ . For males, the mean effects were  $A = 1.370 \cdot 10^{-5}$  and  $B = 2.975$ . The combined-sex results indicated that the mean effects were  $A = 2.768 \cdot 10^{-6}$  and  $B = 3.243$ . The results of a fixed-effects meta-analysis were very similar to the random effects results. The results of the meta-analysis also indicated that there was sexual dimorphism in the length-weight relationship for Pacific blue marlin. This was consistent with the fact that the species exhibits sexual dimorphism in growth in size at age and suggested that there are important differences in the feeding behavior and ecology of female and male blue marlin. While females grow faster and achieve larger lengths at age than males, adult females also achieve greater weights at a given length, on average.

## Introduction

At the April 2012 intercessional meeting of the ISC Billfish Working Group (WG), several sources of information on the probable length-weight relationship of Pacific blue marlin were presented in Sun et al. (2012). The WG concluded that further work was needed to select an appropriate length-weight relationship for conducting a blue marlin stock assessment. In this working paper, I provide a meta-analysis of the available studies that provide standard information on the allometric model of weight as a function of length for female, male, and combined-sex Pacific blue marlin (Wang et al. 2006, Su et al. 2012, Sun et al. 2012). The meta-analysis treated the parameters of the allometric model as effect sizes and combined the information to estimate the mean effect size using the random-effects model. Sample sizes of the available studies were used to weight the effect sizes under a simplifying assumption of homogeneous within-study variances and a rough approximation of the order of magnitude of the between-studies variance. A fixed-effect model was also applied for comparison. The results of the random effects and fixed-effect models were very consistent. Because the available studies were taken from published literature in which effect sizes were effectively being sampled from the distribution of possible effects sizes, it is recommended that the results of the random-effects model be used for stock assessment analyses.

## Methods

Data for the meta-analysis were taken from Wang et al. (2006, Figure 3), Su et al. (2012, Table 1), and Sun et al. (2012, Table 3) and included studies that provided estimates of the parameters (A and B) of the allometric relationship for Pacific blue marlin weight (body weight [BW], kg) as a function of fish length (eye-fork length [EFL], cm), where

(0.1)

$$BW = A \cdot EFL^B$$

This information was based on fitted length-weight relationships for female (Chen 2001, Dai 2002, Su et al. 2012), male (Chen 2001, Dai 2002, Su et al. 2012), and combined-sex (Kume and Joseph 1969, Wares and Sakagawa 1974, Uchiyama and Kazama 2003, Wang et al. 2006) Pacific blue marlin (Table 1).

A random effects meta-analysis model (Borenstein et al. 2009) was applied to estimate mean values of the A and B parameter of the allometric relationship. Under the random effects model, the observed effect size was the length-weight parameter value, which varied from one study to another due to different effect sizes underlying each study and due to random sampling error that was inherent in each study. The observed dispersion reflected both sampling error and the variance of the distribution of the true effects across studies. The observed effect for any study ( $Y_i$ ) was the sum of the grand mean ( $\mu$ ), the deviation of the study's true effect from the grand mean ( $\zeta_i$ ), and the deviation of the study's observed effect from the study's true effect size ( $\varepsilon_i$ )

(0.2)

$$Y_i = \mu + \zeta_i + \varepsilon_i$$

To compute a study's variance for the random effects model one needs to have estimates of both the within-study variance ( $V_{Y_i}$ ) and the variance of the distribution of true effect sizes across studies ( $\tau^2$ ). The weight assigned to each study (indexed by  $i$ ) was  $W_i^*$  where  $V_{Y_i}^*$  was the within-study variance plus the sample estimate of the between-studies variance ( $T^2$ )

$$(0.3) \quad W_i^* = \frac{1}{V_{Y_i}^*} = \frac{1}{V_{Y_i} + T^2}$$

In the absence of information on the within-study variance of length-weight parameters, it was assumed that the within-study variance per sample for each parameter ( $\sigma^2$ ) was equal across studies and that the within-study variance was scaled by the sample size ( $n_i$ ) of each study to obtain

$$(0.4) \quad V_{Y_i} = \frac{\sigma^2}{n_i}$$

In Eqn (1.4), note that we have suppressed an index to denote the A or B parameter for simplicity. For the random effects meta-analysis, it was assumed that the within-study variances for the A and B parameter were on the order of  $\sigma^2 = 1$  and  $\sigma^2 = 0.01$ , respectively. This assumption gave standard deviations on the order of  $\sigma = 1$  and  $\sigma = 0.1$  for the A and B parameters and corresponded to the order of magnitude of observed differences between parameter values reported in the studies (Table 3). Sensitivity of the meta-analysis results to the magnitude of assumed variances was also examined.

Given the within-study variances, a sample estimate of the between-studies variance with a total of  $K$  studies based on the method of DerSimonian and Laird was

$$(0.5) \quad T^2 = \frac{Q - K + 1}{C}$$

where  $K$  was the number of studies and  $Q$  and  $C$  were constants that depend on the study weights and effect sizes (see Borenstein et al. 2009, pp. 72-73). Given the study weights, the mean effect size, denoted by  $M^*$ , was computed as a weighted mean of the individual study effects

$$(0.6) \quad M^* = \frac{\sum_{i=1}^K W_i^* Y_i}{\sum_{i=1}^K W_i^*}$$

The sensitivity of the mean effects to the assumed within-study variance was assessed by recomputing the meta-analysis weights for the A and B parameters with variance of  $\frac{1}{100}\sigma^2$  and  $100\sigma^2$ . This was done to show the effects of a hundred-fold decrease or increase in the within-study variance on the mean effects.

A fixed-effect meta-analysis was also conducted for comparison. Under the fixed-effect model, the assumption that the within-study variance per sample was equal across studies implied that the study weights were equal to the sample sizes of the studies. In this case, the mean M, or common effect, was simply the average of the study effects weighted by sample size

$$(0.7) \quad M = \frac{\sum_{i=1}^K n_i Y_i}{\sum_{i=1}^K n_i}$$

## Results

Random effects meta-analysis results (Table 1) indicated that the mean effects for females were  $A = 1.844 \cdot 10^{-5}$  and  $B = 2.956$ . For males, the mean effects were  $A = 1.370 \cdot 10^{-5}$  and  $B = 2.975$ . The combined-sex results indicated that the mean effects were  $A = 2.768 \cdot 10^{-6}$  and  $B = 3.243$ . The female and male results showed that there was sexual dimorphism in the estimated parameters of the allometric model (Figure 1). The male scale parameter A was about 26% lower than the female A value and the male exponent parameter B was about 1% higher than the female B value. In addition, the combined-sex allometric model was more similar to the male model than the female model (Figure 1). It was also notable that the scale and exponent parameters of the combined-sex model differed from those of the female and male models (Table 1). The fixed-effects meta-analysis results were very similar to the random effects results (Table 1, Figure 1 and 2). Under the fixed-effect model, the percent changes in the female A and B parameters were -0.4% and 0.0% relative to the random effect model. For males and combined-sex blue marlin, there were no changes in A and B under the fixed-effect model relative to the random effect model. Overall, the choice of meta-analysis approach had a negligible effect on the results.

The sensitivity analysis for the assumed scale of the within-study variance showed that results were not sensitive to a hundred-fold decrease or increase in this variance. The mean effects for both females and males with a variance of  $\frac{1}{100}\sigma^2$  were virtually

identical to those with a variance of  $\sigma^2$  and the percent changes with the lower variance were 0% and 0% for A and B parameters for both sexes. Similarly, the results for females, males, and combined sexes with a variance of  $100\sigma^2$  were virtually identical with percent changes of less than 0.1% for both A and B parameters. Overall, the meta-analysis results were robust to changes on the order of a hundred-fold decrease or increase in the within-study variance.

## Discussion

Results of the meta-analysis were robust to the selection of a random effects or a fixed-effect model for the analysis. However, the available studies were taken from published literature in which effect sizes are effectively being sampled from the distribution of possible effects sizes. As a result, the assumptions of the random effects model were most consistent with the observed information and it is recommended that the random effects results be used for stock assessment analyses.

The results of the meta-analysis also indicated that there was sexual dimorphism in the length-weight relationship for Pacific blue marlin. This was consistent with the fact that the species exhibits sexual dimorphism in growth in size at age (Skillman and Yong 1976, Hill 1986, Chen 2001, Dai 2002, Shimose 2008) and suggested that there are important differences in the feeding behavior and ecology of female and male blue marlin. While females grow faster and achieve larger lengths at age than males, adult females also achieve greater weights at a given length, on average. The onset of differences in the length-weight relationship of females and males appears to correspond to the median size at maturity of females of about 180 cm (Sun et al. 2012, see Figure 4) and this change in relative body mass at length is likely an adaptation of females to maximize their lifetime reproductive output.

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Table 1. Random effects and fixed-effect meta-analyses of length (EFL, cm)-weight (BW, kg) parameters (A and B) for female, male, and combined-sex Pacific blue marlin as a function of sample size (n) and within-studies variance ( $\sigma^2$ ).

<b>Female Blue Marlin Length-Weight Parameters</b>					
<b>Study</b>	<b>A (units are <math>10^{-5}</math>)</b>	<b>B</b>	<b>n</b>	<b>Weight for A</b>	<b>Weight for B</b>
Chen 2001	1.0	2.996	926	930.57	926.02
Dai 2002	6.0	2.7002	257	261.57	257.02
Su 2012	1.427	2.996	717	721.57	717.02
<b>Random-Effects Weighted Mean M*</b>	<b>1.844</b>	<b>2.956</b>	1900	1913.70	1900.05
<b>Fixed-Effect Weighted Mean M</b>	<b>1.837</b>	<b>2.956</b>			
Q	5223.21	19.45			
C	1143.36	1143.36			
T2	4.57	0.02			
$\sigma^2$	1.00	0.01			
<b>Male Blue Marlin Length-Weight Parameters</b>					
<b>Study</b>	<b>A (units are <math>10^{-5}</math>)</b>	<b>B</b>	<b>n</b>	<b>Weight for A</b>	<b>Weight for B</b>
Chen 2001	2.0	2.883	666	666.29	666.01
Dai 2002	1.0	2.9763	418	418.29	418.01
Su 2012	1.116	3.033	1043	1043.29	1043.01
<b>Random-Effects Weighted Mean M*</b>	<b>1.370</b>	<b>2.975</b>	2127	2127.88	2127.02
<b>Fixed-Effect Weighted Mean M</b>	<b>1.370</b>	<b>2.975</b>			
Q	388.85	9.15			
C	1324.87	1324.87			
T2	0.29	0.01			
$\sigma^2$	1.00	0.01			
<b>Combined-Sex Blue Marlin Length-Weight Parameters</b>					
<b>Study</b>	<b>A (units are <math>10^{-6}</math>)</b>	<b>B</b>	<b>n</b>	<b>Weight for A</b>	<b>Weight for B</b>
Kume 1969	5.5565	3.0888	11	11.94	11.01
Wares 1974	2.0417	3.318	57	57.94	57.01
Uchiyama 2003	1.3	3.43	32	32.94	32.01
Wang 2006	2.79	3.24	2548	2548.94	2548.01
<b>Random-Effects Weighted Mean M*</b>	<b>2.768</b>	<b>3.243</b>	2648	2651.76	2648.03
<b>Fixed-Effect Weighted Mean M</b>	<b>2.767</b>	<b>3.243</b>			
Q	185.79	1.72			
C	194.56	194.56			
T2	0.94	0.01			
$\sigma^2$	1.00	0.01			



Figure 1. Results of random effects meta-analysis of female, male, and combined-sex Pacific blue marlin length-weight parameters.

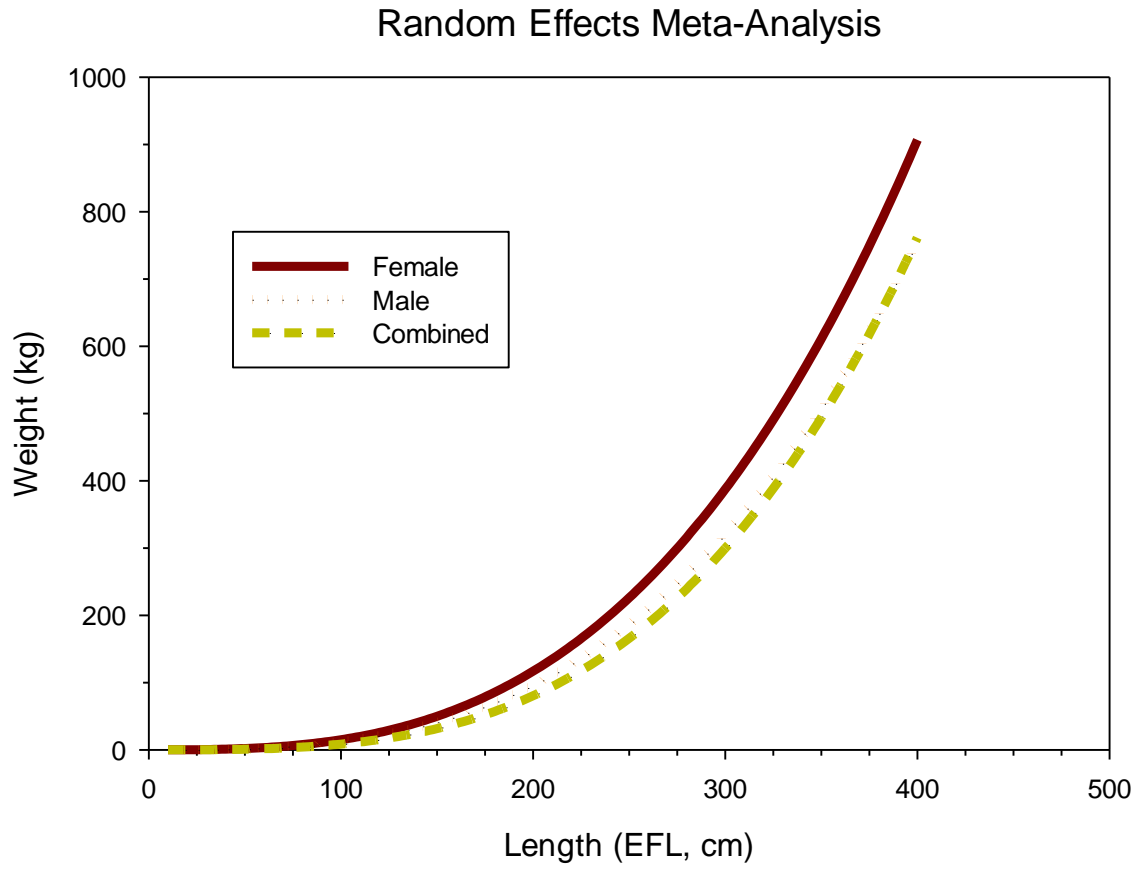


Figure 2. Results of fixed effect meta-analysis of female, male, and combined-sex Pacific blue marlin length-weight parameters.

