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Development of an Informative Prior for 'r', the intrinsic rate of population increase, for striped marlin (*Tetrapturus audax*)

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Abstract

This paper documents the development of an informative prior for the parameter describing the intrinsic rate of population increase (r) which is used in surplus production stock assessment models. The approach is based on demographic methods and draws heavily from the literature on striped marlin and other related species. By defining a range of likely values for r, robust estimation of other parameters, including stock assessment reference points, by the proposed Bayesian surplus production model is facilitated.

Introduction

It has been agreed that a Bayesian surplus production model will be applied to available catch and effort at the November 2005 striped marlin stock assessment workshop. The selected model (ICCAT 2003a) has been applied to several large pelagic species including white marlin (*Tetrapturus albidus*) in the 2002 ICCAT assessment (Babcock & McAllister 2003). In this assessment, and other assessments applying this model, a high correlation between model parameters r, the intrinsic rate of population increase, and K, population carrying capacity, was observed (McAllister et al. 2001, ICCAT 2005, Clarke & McAllister 2005). This problem, which often occurs when there is a lack of contrast in the time series of catchdata, results in difficulties in choosing between sets of r and K values to fit the model and leads to highly uncertain conclusions.

One solution is to focus estimation of either r or K on a range of likely values by developing an informative prior probability distribution for either parameter. McAllister et al. (2001) outlines 3 methods for developing an informative prior for r including generation time, Leslie matrix and Euler-Lotka approaches. This paper is based on implementation of the Euler-Lotka method as illustrated in Babcock & McAllister (2003).

Methods and Model Validation

The life history parameters necessary to estimate r were defined by an alternative form of the Euler-Lotka equation provided by Myers et al. (1997):

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$$\left(e^{r}\right)^{a} - \exp(-M)\left(e^{r}\right)^{a-1} - \alpha = 0 \qquad \text{Eq. 1}$$

where a is the age at maturity, M is the instantaneous adult natural mortality rate, and α is maximum annual reproductive rate (or slope of the stock-recruit relationship at the origin).

Using WinBUGS software (Anon. 2004), 500 sets of values for each parameter were randomly drawn from distributions specified as follows:

$$a = F(x) = \begin{cases} x & if\ 1 \le x \le 7 \\ 0 & otherwise \end{cases} \qquad \text{Eq. 2}$$

$$M \sim \log N(0.1, 0.16) \qquad \text{Eq. 3}$$

$$\log \alpha \sim N(0.818, 1.37) \qquad \text{Eq. 4}$$

These values were selected to duplicate the analysis in Babcock & McAllister (2003) for validation purposes. As the exact values for log α were not provided, they were calculated directly from Myers (1999). Values for a were rounded to the nearest integer. The parameter sets resulting from the WinBUGS iterations were used to calculate r with the Excel Solver function and a Visual Basic macro applying the constraint $0.01 \le r \le 1.2$. The median and variance of the 500 values of r were then computed and compared to Babcock & McAllister's (2003) white marlin results. This analysis produced a median estimate of 0.400 and a standard deviation of 0.297 which was closely comparable to the published results of 0.405 (median) and 0.300 (standard deviation).

A similar validation exercise was conducted to attempt to duplicate the results of a subsequent informative prior analysis conducted at the ICCAT 2002 white marlin assessment meeting (ICCAT 2003b). For this exercise the following distributions were implemented:

$$a \sim \log N(3.5, 0.1)) \qquad \text{Eq. 5}$$

$$M \sim \log N(0.1, 0.25) \qquad \text{Eq. 6}$$

$$\log \alpha \sim N(0.49, 1.14) \qquad \text{Eq. 7}$$

It is noted that the standard deviation, and thus variance, for the distribution of age at maturity (a) was given as 1. However, this produced a distribution with a 97.5^{th} percentile of 24. In order to produce a distribution like that described in ICCAT (2003) (Figure E1-b and the text), a variance of 0.1 was applied which produced a distribution with a 97.5^{th} percentile of 6.4. The median and standard deviation of r were computed as 0.427 and 0.262, respectively. These estimates were very similar to the median (0.43) and standard deviation (0.22) produced in the ICCAT workshop (ICCAT 2003).

Basis for Striped Marlin Parameter Estimates

In order to develop an informative prior for striped marlin, a literature search was conducted for relevant demographic parameters for this species. The parameter for age at maturity (a) was taken from Au (1998) which estimated age at 50% maturity as 4 for striped marlin, and from 3 to 5 for swordfish (*Xiphias gladius*), blue marlin (*Makaira mazara*) and sailfish (*Istiophorus platypterus*). This information formed the basis for a normally distributed random variable with mean 4.0 and variance 0.26:

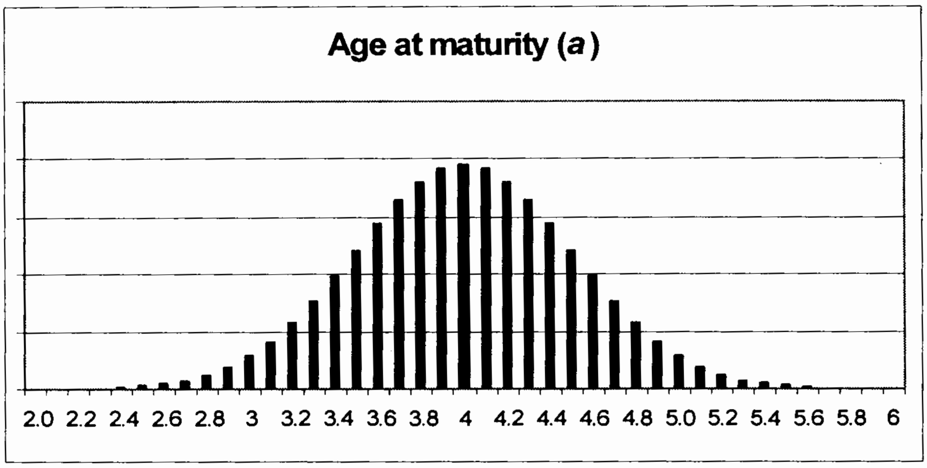

Figure 1. Selected probability distribution for striped marlin age at maturity, $a \sim N(4, 0.26)$, based on parameters from Au (1998).

Natural mortality (M) was estimated by Boggs (1989) based on the von Bertalaffy growth rate coefficient (K) using the formula of Murphy & Sakagawa (1977):

$$M = 1.879K \qquad \text{Eq. 8}$$

This formula produced estimates of M of 0.49 for unsexed striped marlin (based on Koto (1963)), and 0.79 for males and 1.33 for females (based on Skillman and Yong (1976), all age groups, model 1). Estimates for the Pacific swordfish, blue marlin and sailfish ranged from 0.21 to 0.90.

In contrast, M was estimated by Au (1998) from an estimate of maximum reproductive age (ω) using the formula of Hoenig (1983):

$$\ln(M) = 1.46 - 1.01 \cdot \ln(\omega) \qquad \text{Eq. 9}$$

Au's estimate of M for striped marlin was 0.47 and the M values for the related species ranged from 0.21 to 0.53.

This information is represented by a log normally distributed random variable with median 0.5 and variance 0.25:

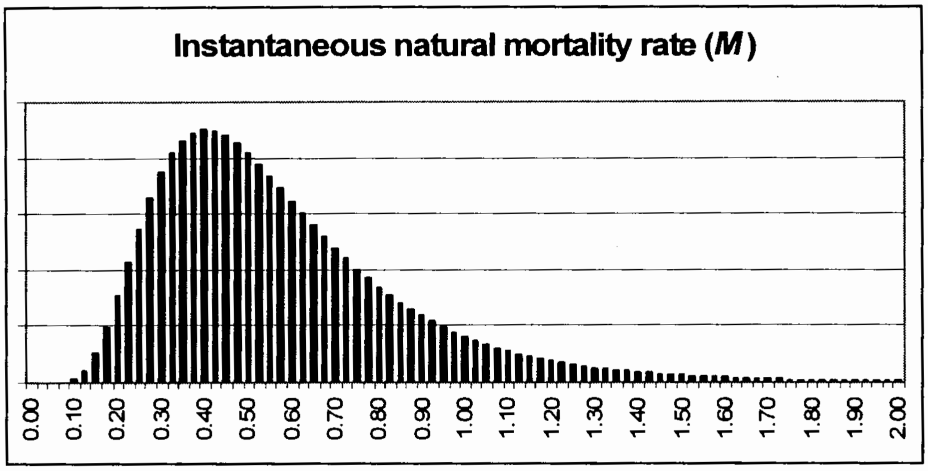

Figure 2. Selected probability distribution for striped marlin instantaneous natural mortality, M~logN(0.5, 0.25), based on parameters from Boggs (1989) and Au (1998).

Based on a lack of information in the literature regarding theoretical maximum values of α for striped marlin, the value of α used in the white marlin assessment was adopted by default. As there was no basis for concluding that the estimate for striped marlin should be any more uncertain than the estimate for white marlin, the variance of the distribution was also left unchanged. Thus, a normally distributed random variable with mean 0.49 and variance 1.14 was specified and produced the following distribution:

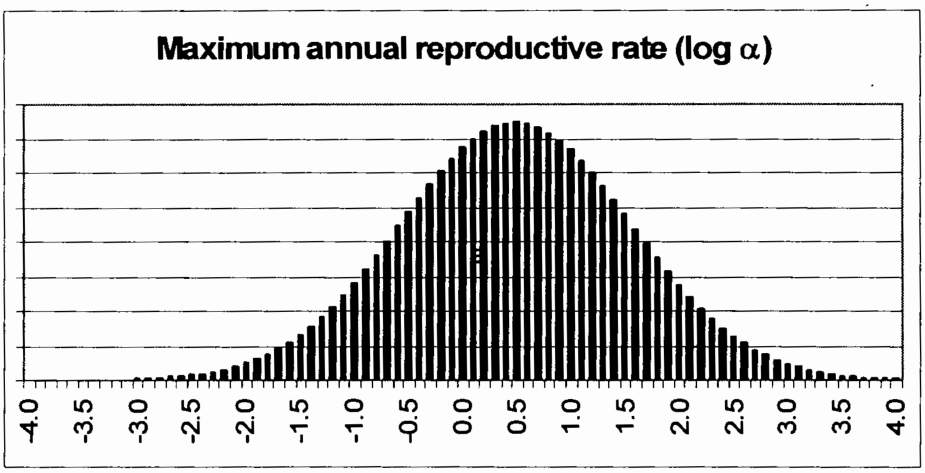

Figure 3. Selected probability distribution for striped marlin maximum annual reproductive rate, log α~N(0.49, 1.14), adopted without adjustment from the white marlin Bayesian prior development exercise (ICCAT 2003b).

Results

Five hundred values of *a*, *M* and α were drawn from the probability distributions specified above and then used to solve for *r*. Lower boundary conditions were encountered for 47 of the iterations, i.e. r=0.01. The mean, median and standard deviation in *r* calculated from the iterations that did not encounter the boundary condition were 0.322, 0.302 and 0.200, respectively.

A log normal prior probability distribution for *r* calculated using the mean (0.322) as the log normal median and a log standard deviation corresponding to the standard deviation (0.2) produced a 95% probability interval of 0.22 to 0.48 (Figure 4).

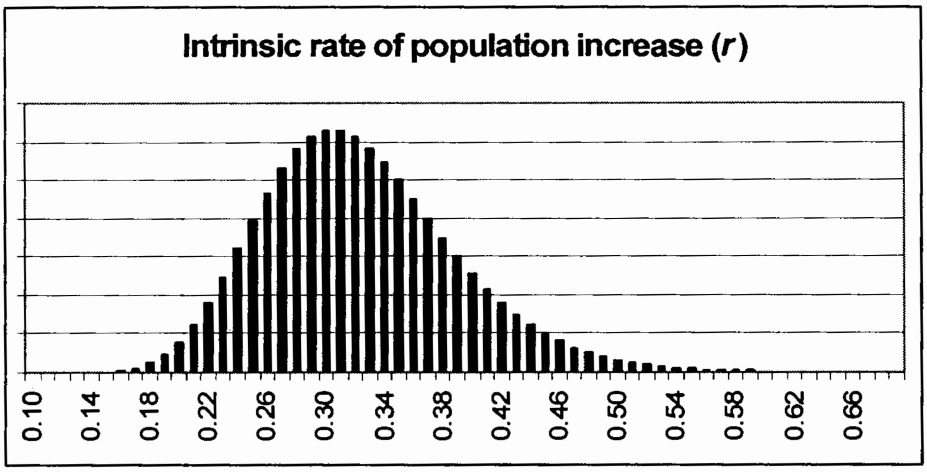

Figure 4. Estimated probability distribution for the intrinsic rate of population increase (*r*), $r \sim \log N(0.322, 0.04)$, using the parameter distributions shown in Figures 1 through 3.

The large number of iterations which encountered boundary conditions in the optimalization step (9%) indicates that there are likely to be some problems in the specification of the probability distributions for the input parameters. Examination of the iterations which encountered the lower boundary conditions indicated that the cause was likely to be low values of α. The least species-specific information was available for the specification of α and thus the appropriateness of the input for striped marlin is uncertain. However, the same distribution provided reasonable values for white marlin, and there is no compelling evidence to suggest that white marlin and striped marlin have substantially different demographic traits.

The large uncertainty in the estimates of *M* was suspected to also contribute to the downward bias in *r*. However, when the standard deviation in *M* was arbitrarily reduced by 50%, approximately the same number of iterations encountered the boundary condition and the estimates for *r* were nearly identical.

Discussion

The informative prior distribution for *r* generated in this analysis represents a first attempt to incorporate existing demographic information on striped marlin and related species into the parameters of the surplus production model. The resulting estimates of *r* appear to be reasonable and thus can be proposed for use at the upcoming striped marlin stock assessment workshop.

If participants are concerned with any of the parameter inputs, these can either be modified and the analysis re-run, or the variance can be increased to account for non-specific uncertainty. A ranking exercise of the values of α provided Myers (1999), similar to that at the white marlin workshop, or exploration of other quantitative methods to derive more appropriate values of α, may be warranted. High uncertainty in M may also be adjusted on the basis of expert judgment but as illustrated by the sensitivity analysis presented above, moderate changes in M are unlikely to have a strong effect on the distribution of r values. Therefore, the greatest improvements in the estimation of r are likely to derive from efforts to better specify input values of α for striped marlin and such efforts are thus recommended for future work.

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